Exercise 317

For the following problems, state the domain and range of the given functions:

$$f = x^2 + 2x - 3, \qquad g = \ln(x - 5), \qquad h = \frac{1}{x + 4}$$

$$g \circ f$$
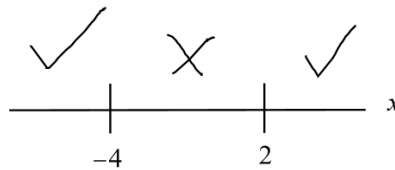

Solution

Evaluate $h \circ f$.

$$g \circ f = g(f(x)) = g(x^2 + 2x - 3) = \ln((x^2 + 2x - 3) - 5) = \ln(x^2 + 2x - 8)$$

$g \circ f$ is a logarithmic function, and the one thing to know about logarithmic functions is that the argument must be positive.
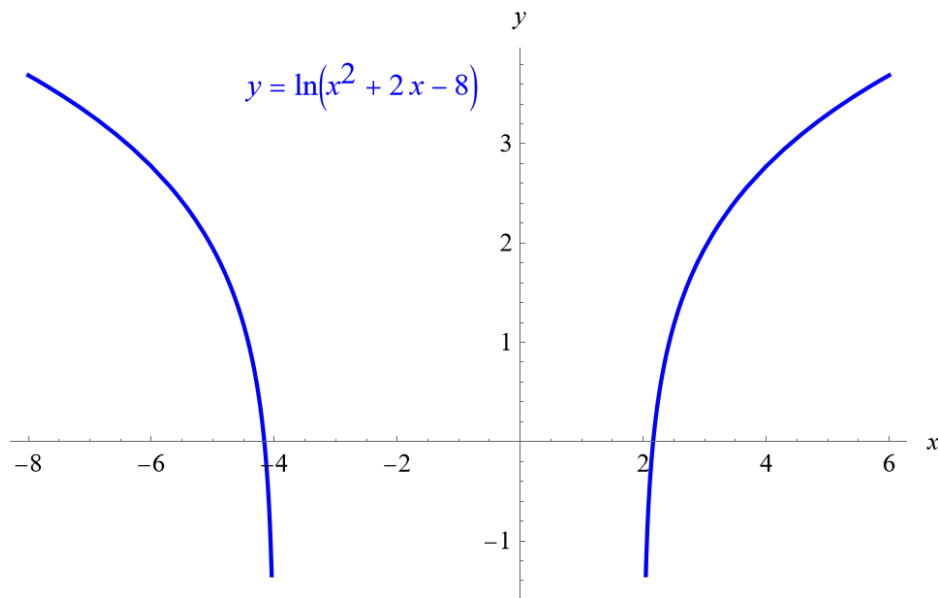
$$x^2 + 2x - 8 > 0$$

$$(x + 4)(x - 2) > 0$$

The critical points are $x = -4$ and $x = 2$. Test the inequality for values of x in the intervals between these critical points.

Therefore, the domain is $(-\infty, -4) \cup (2, \infty)$. Below is a graph of $g \circ f$ versus x.

The logarithmic function takes on all y-values. Therefore, the range is $\{y \mid -\infty < y < \infty\}$.